Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

of *John D. Williams' Elementary Treatise on Algebra*, Boston, 1840. The solution there given is satisfactory, but extremely lengthy, and very complex. The results obtained are as follows:

The sides of the first triangle	18601944; 13951458; 23252430.
" " " " second triangle	13999464; 18559223; 23247145.
" " " " third triangle	18515584; 14048388; 23241860.

Common perimeter = 55805832.

Radii of the inscribed circles are 4650486; 4655771; 4661056.

SOLUTION OF A PROBLEM.

BY PROF. DAVID TROWBRIDGE, WATERBURGH, N. Y.

Required the Sum of the Products of n Quantities taken m and m together, no two products having the same Factors.

Let the n quantities be $r_1, r_2, r_3, \dots r_n$. Let S_m be the required sum. Also let

$$S_1 = R_1 = r_1 + r_2 + \dots + r_n,\ R_2 = r_1^2 + r_2^2 + \dots + r_n^2, \dots$$
$$R_m = r_1^m + r_2^m + \dots + r_n^m \quad (1).$$

Now

$$r_1(r_2 + r_3 + \dots + r_n) + r_2(r_1 + r_3 + \dots + r_n) + \dots$$
$$+ r_n(r_1 + r_2 + \dots + r_{n-1}) = r_1(R_1 - r_1) + r_2(R - r_2) + \dots$$
$$+ r_n(R_1 - r_n) = R_1^2 - R_2 = R_1S_1 - R_2.$$

But we have evidently taken each product twice, so that we shall consequently have

$$2S_2 = R_1S_1 - R_2 \dots\dots\dots\dots\dots\dots (2).$$

$$r_1(r_2r_3 + r_2r_4 + \dots) + r_2(r_1r_3 + r_1r_4 + \dots) + \dots$$
$$+ r_n(r_1r_2 + r_1r_3 + \dots) = r_1[S_2 - r_1(R_1 - r_1)]$$
$$+ r_2[S_2 - r_2(R_1 - r_2)] + \dots + r_n[S_2 - r_n(R_1 - r_n)]$$
$$= R_1S_2 - R_1R_2 + R_3 = R_1S_2 - R_2S_1 + R_3.$$

But we have now taken each product three times. We hence have

$$3S_3 = R_1S_2 - R_2S_1 + R_3 \dots\dots\dots\dots\dots\dots (3).$$

$$r_1(r_2r_3r_4 + r_2r_3r_5 + \dots) + r_2(r_1r_3r_4 + r_1r_3r_5 + \dots) + \dots$$
$$= r_1\left\{S_3 - r_1[S_2 - r_1(R_1 - r_1)]\right\} + r_2\left\{S_3 - r_2[S_2 - r_2(R_1 - r_2)]\right\}$$
$$+ \dots = R_1S_3 - R_2S_2 + R_3R_1 - R_4 = R_1S_3 - R_2S_2 + R_3S_1 - R_4 \dots$$
$$\therefore 4S_4 = R_1S_3 - R_2S_2 + R_3S_1 - R_4 \dots\dots\dots (4.)$$

We now easily see that the general equation will be

$$mS_m = R_1S_{m-1} - R_2S_{m-2} + R_3S_{m-3} - \dots + (-1)^{m-2}R_{m-1}S_1 + (-1)^{m-1}R_m \dots\dots\dots (5.)$$

The general equation of the n^{th} degree is

$$x^n + A_1x^{n-1} + A_2x^{n-2} + \dots + A_n \dots\dots\dots (6.)$$

Let the n roots of this equation be $r_1, r_2, \dots r_n$, so that (6) becomes (not regarding the signs of the roots)

$$(x + r_1)(x + r_2)(x + r_3)\dots(x + r_n) = x^n + S_1x^{n-1} + S_2x^{n-2} + \dots (7).$$

That is

$$S_1 = A_1,\ S_2 = A_2,\ S_3 = A_3,\ \text{\&c.}$$

By means of equation (2) we have

$$R_2 = S_1R_1 - 2S_2 = S_1^2 - 2S_2 = A_1^2 - 2A_2 \dots\dots\dots (8).$$

From (3)

$$R_3 = 3S_3 - R_1S_2 + R_2S_1 = 3A_3 - A_1A_2 + A_1(A_1^2 - 2A_2)$$
$$= 3A_3 - 3A_1A_2 + A_1^3 \dots\dots\dots (9).$$

In a similar manner we find

$$R_4 = A_1^4 - 4A_1^2A_2 + 2A_2^2 + 4A_1A_3 - 4A_4 \dots\dots\dots (10).$$

It is also evident that we could easily deduce the values of R_5, R_6, &c. We have therefore found, in this way, the sum of the squares, the sum of the cubes, the 4th powers, and so on, of the roots of an equation in terms of the coefficients of the equation. These powers may extend up to the n^{th}, but not beyond, since $S_n = r_1r_2r_3 \dots\dots r_n$, and we cannot have a product with any more factors than n.

From (5) we see that

$$r_1 r_2 r_3 \ldots r_n = \tfrac{1}{n}[R_1 S_{n-1} - R_2 S_{n-2} + \ldots + (-1)^{n-2} R_{n-} S_1 + (-1)^{n-1} R_n].$$

$n = 2, 3, 4,$ &c.,

$$r_1 r_2 = \tfrac{1}{2}[(r_1 + r_2)^2 - r_1^2 - r_2^2];\ r_1 r_2 r_3 = \tfrac{1}{3}[\tfrac{1}{2}(r_1 + r_2 + r_3)^3 - \tfrac{1}{2}(r_1 + r_2 + r_3)(r_1^2 + r_2^2 + r_3^2) - (r_1 + r_2 + r_3)(r_1^2 + r_2^2 + r_3^2) + r_1^3 + r_2^3 + r_3^3] = \tfrac{1}{3}[\tfrac{1}{2}(r_1 + r_2 + r_3)^3 -- \tfrac{3}{2}(r_1 + r_2 + r_3)(r_1^2 + r_2^2 + \tfrac{2}{3}) + r_1^3 + r_2^3 + r_3^3].$$

These transformations show how easily we may derive results by a general process, that would otherwise be difficult.

* * *

DEMONSTRATION OF TWO OF PROF. PEIRCE'S PROPOSITIONS.

BY PROF. M. L. COMSTOCK, OF KNOX COLLEGE, ILL.

In the *Mathematical Monthty* (Runkle's), No. 1, Vol. 1, Prof. Peirce gave a set of "Propositions on the Distribution of Points on a Line." They were subsequently solved by Prof. W. P. G. Bartlett and R. J. Adcock. I send you a solution of two of them by a method presumably applicable to the others.

Prop. 1.—"If two points are taken in all possible relative positions, upon a given line, their distance apart in one-half the whole number of possible positions, is less than .29289 of the length of the line."

Prop. 2.—"The number of positions in which the distance between the points is less than half the length of the line is $\frac{3}{4}$ of the whole number of possible positions."

If a = the length of the line, and x = the shortest distance between two points, it readily appears that

$\frac{a}{x} =$ number of positions at the distance x,

$\frac{a}{x} - 1 =$ " " " $2x$.

$\frac{a}{x} - 2 =$ " " " $3x$,

⋮

$2 =$ " " " $a - x$,

$1 =$ " " " a,